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Higher-Order Voronoi Diagram of Line Segments

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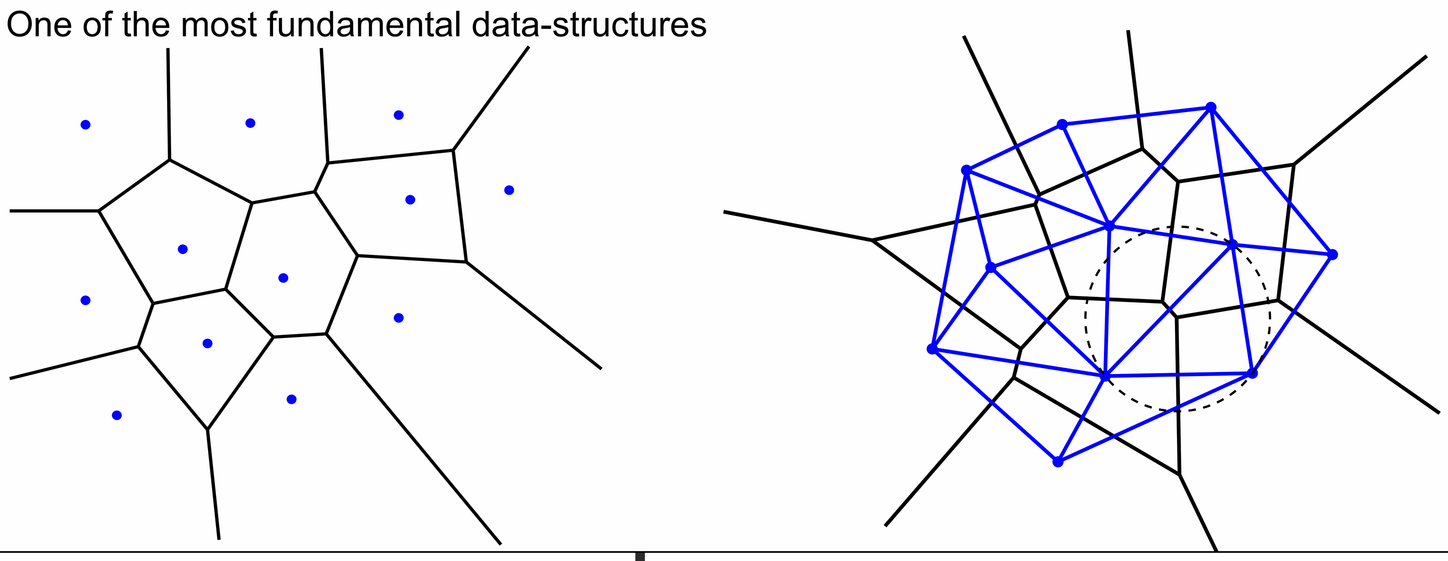
Abstract

The Voronoi diagram is a powerful geometric object that has found lots of applications in different areas. Lots of generalizations of the Voronoi diagram have been studied, however, the higher-order Voronoi diagram of line segments has been still largely ignored in the literature. The goal is to fill the gap. We consider some of the most studied Voronoi diagrams (the nearest neighbor Voronoi diagram of points, the order-k Voronoi diagram of points, etc) and the farthest line segment Voronoi diagram, as the corner cases of the higher-order Voronoi diagram of line segments. And we investigate which of the properties of the corner cases can be generalized to the higher-order Voronoi diagram of line segments.

Nearest Neighbor Voronoi Diagram of Points

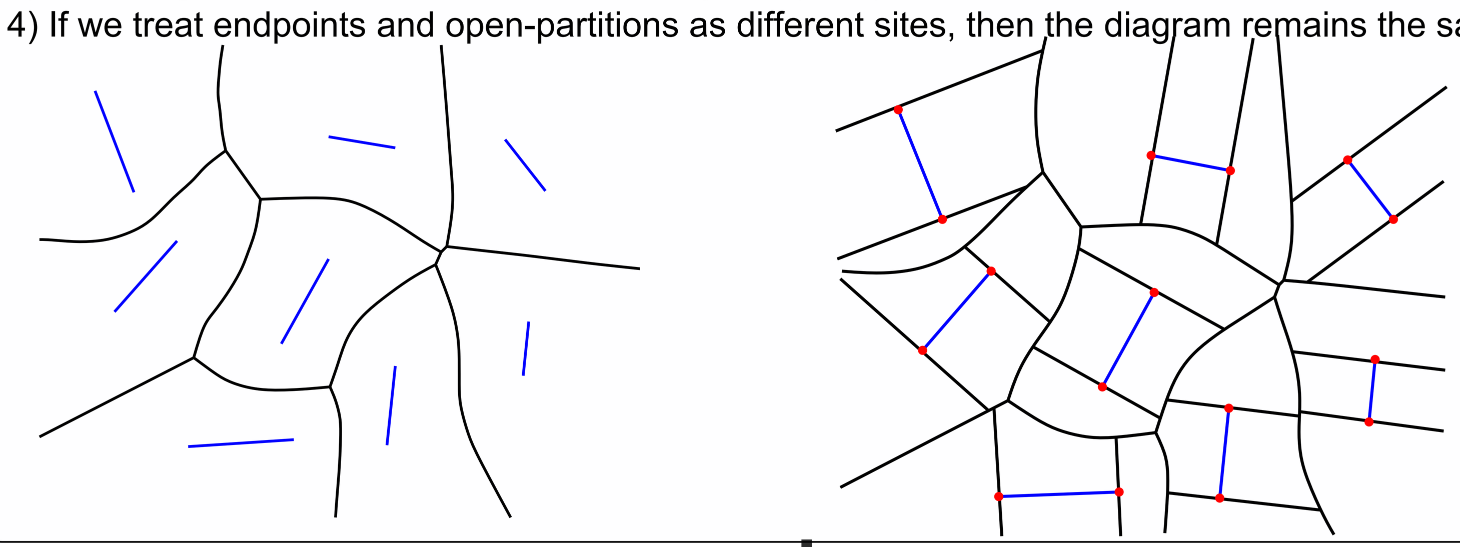
1) Structural complexity is O(n)
2) Dual to the Delaunay triangulation
3) Has relation with a convex hull, through the lift-up transformation
4) One of the most fundamental data-structures



generalization

Nearest Neighbor Voronoi Diagram of Line Segments

1) Structural complexity is O(n)
2) If the segments form a PSLG the complexity remains O(n)
3) If the segments do intersect the complexity is O(m+n), where m is the number of intersections
4) If we treat endpoints and open-partitions as different sites, then the diagram remains the same

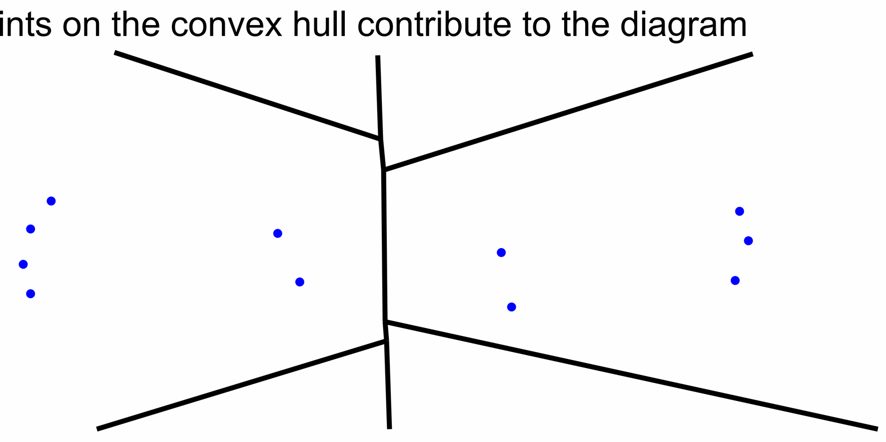


generalization

generalization

Farthest Voronoi Diagram of Points

1) The structural complexity is O(n)
2) All the regions are unbounded
3) The diagram is a tree
4) Only the points on the convex hull contribute to the diagram

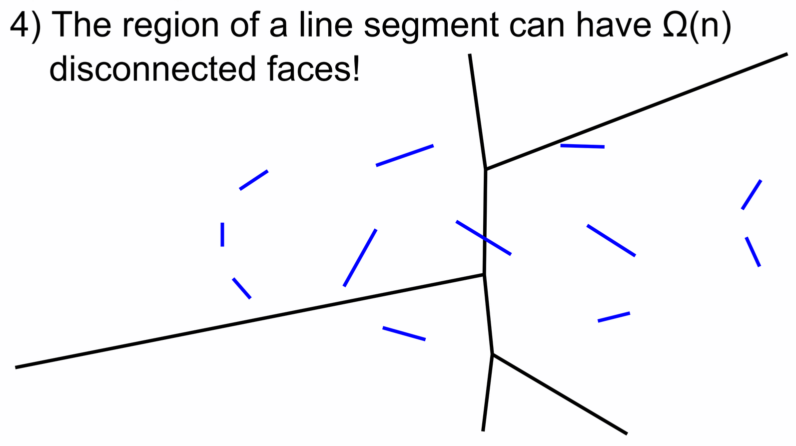


generalization

generalization

Farthest Voronoi Diagram of Line Segments

1) The structural complexity is O(n)
2) All the regions are unbounded
3) The diagram is a tree
4) The region of a line segment can have Ω(n) disconnected faces!
5) The regions of the diagram are not represented by a convex hull of segments
6) If we treat endpoints and open-partitions as different sites, then only the endpoints contribute to the structure of the diagram. Therefore the diagram coincides with the farthest Voronoi diagram of the endpoints.

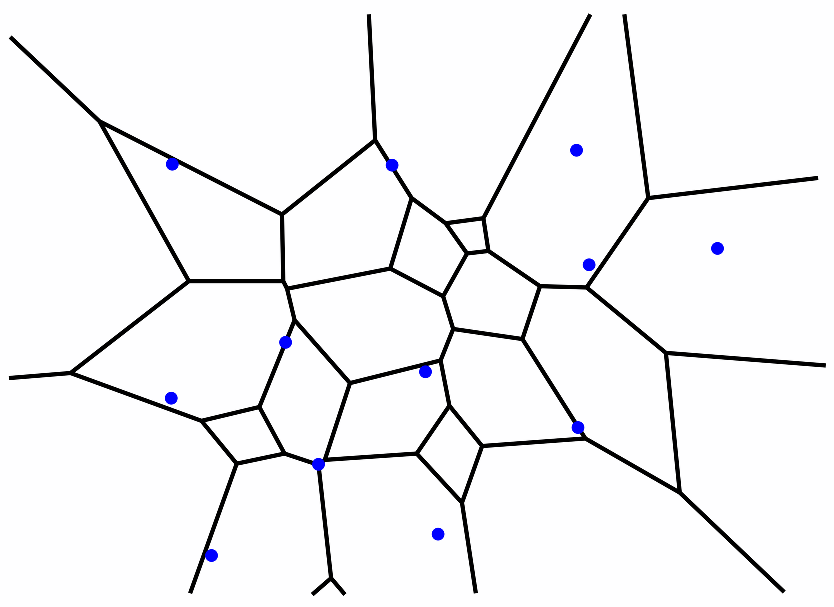


generalization

generalization

Order-k Voronoi Diagram of Points

1) The structural complexity is O(k(n-k))
2) Nearest Neighbor Voronoi diagram of points is a corner case, for k=1
3) Farthest Voronoi diagram of points is a corner case, for k=n-1
4) Dual to the k-nearest neighbor Delaunay graph
5) Has a relation with arrangements and convex hulls

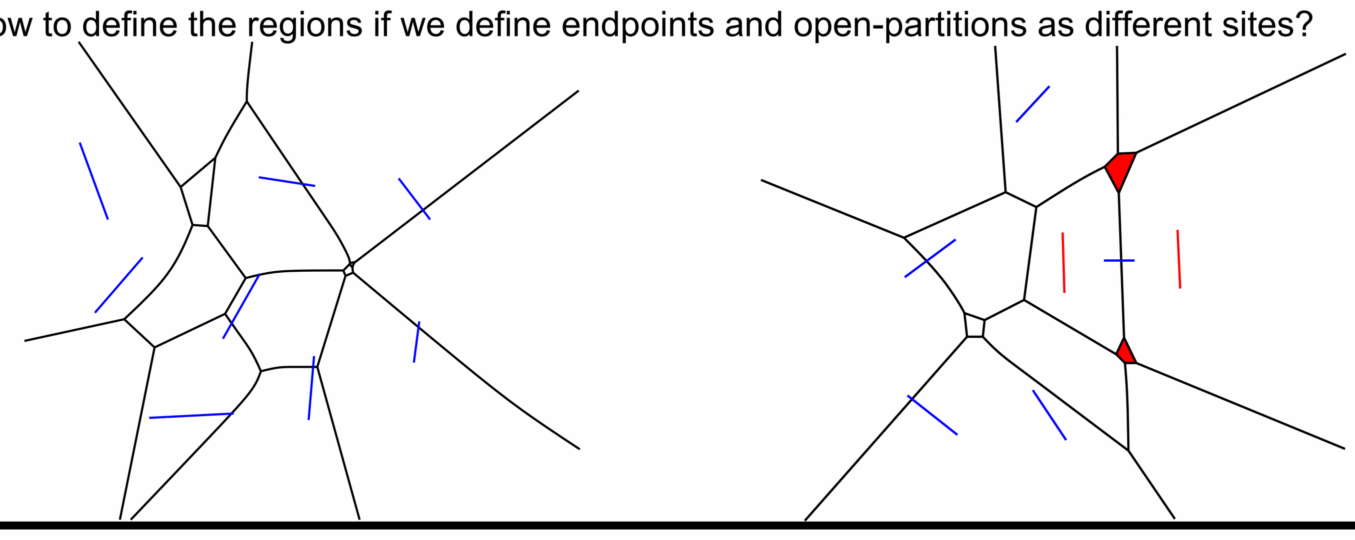


generalization

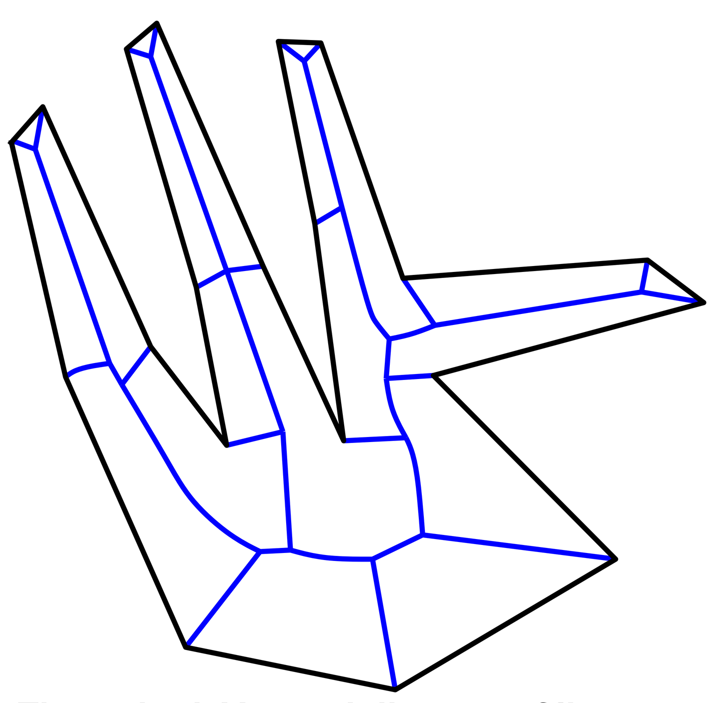
Order-k Voronoi Diagram of Line Segments (Current Research)

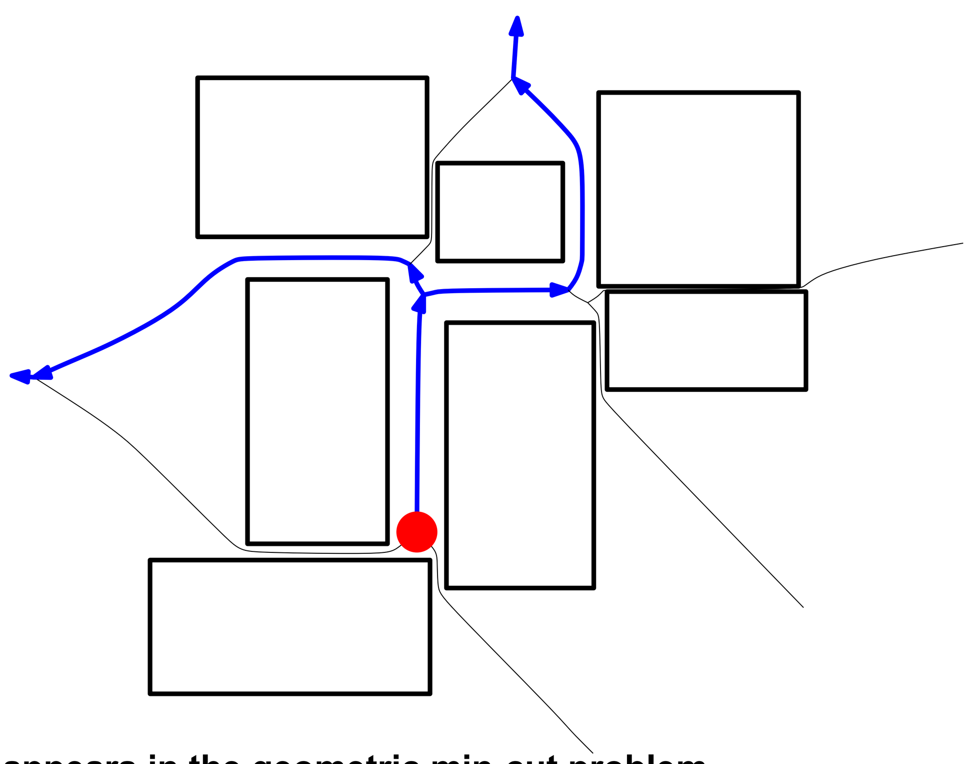
1) Nearest Neighbor Voronoi diagram of segments is a corner case, for k=1
2) Farthest Voronoi diagram of segments is a corner case, for k=n-1
3)The region of a line segment can have Ω(k) disconnected faces!
4) A region can have at most O(k) disconnected unbounded faces

Open problems:
1) Is the structural complexity O(k(n-k))?
2) How to define the regions if the segments form a PSLG?
3) How to define the regions if we define endpoints and open-partitions as different sites?



Applications





The order-k Voronoi diagram of line segments appears in the geometric min-cut problem that can model, among others, the VLSI critical area extraction problem.

The special cases of the order-k Voronoi diagram of line segments are also used in the applications:
1) The nearest neighbor Voronoi diagram of line segments can be also applied to build a medial axis of a polygonal object.
2) The nearest neighbor Voronoi diagram of a set of objects can be applied in motion planning, to find a safe path for a moving object.
3) The nearest neighbor Voronoi diagram of a set of objects can be also applied in a growth prediction field.

Definitions

The nearest neighbor Voronoi diagram of a set of objects S in the plane, called sites, is a partitioning of the plane into regions, such that the Voronoi region of a site s is the locus of points closer to s than to any other site.

The order-k Voronoi diagram of a set of sites S in the plane is a partitioning of the plane into regions, such that each point within a fixed region has the same closest k sites.

The farthest Voronoi diagram is a generalization of the nearest neighbor Voronoi diagram. Each Voronoi region in the farthest Voronoi diagram of sites is a locus of points which are farthest from the associated site than from any other site.

The Delaunay triangulation of the point set S is a triangulation of S such that no point of S is inside the circumcircle of any triangle, where triangulation is a subdivision of the convex hull of points S into triangles

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